OFDM Inter-Carrier Interference Due To Radio Frequency Synthesizer Phase Noise

Vítor Fialho\textsuperscript{ab}, Fernando Azevedo\textsuperscript{ac}, Fernando Fortes\textsuperscript{ac}, Manuela Vieira\textsuperscript{ab}

\textsuperscript{a}Instituto Superior de Engenharia de Lisboa – ISEL, Rua Conselheiro Emídio Navarro, 1 1959-007-Lisbon, Portugal
\textsuperscript{b}Universidade Nova de Lisboa – FCT– DEE, Monte da Caparica 2829-516, Lisbon, Portugal
\textsuperscript{c}Instituto das Telecomunicações, Av. Rovisco Pais, 1 1049 001 Lisbon, Portugal

Abstract

This work presents the effects of radio-frequency Local Oscillator non-idealities on OFDM inter-carrier interference. The synthesizer has the main role on modulation and demodulation process of base band IQ signals to radio frequency channels which need to fulfill several requirements imposed by standards specifications. Phase and frequency stability are crucial, since the synchronization of the entire system depends on the accuracy of this circuit. The discussed OFDM system is based on a simulation scenario including radio-frequency channel conversion considering Local Oscillator with configurable phase noise power and bandwidth. This feature allows the study of global system based on Local Oscillator output spectrum. Sub-channel spacing, cyclic redundancy and pulse shaping are configurable. The obtained results indicate that free run oscillator phase noise characterization is not appropriate when the Local Oscillator in the radio-frequency front end is based on synthesizer architecture. System evaluation is based on error vector magnitude. The relation of this metric with the Local Oscillator phase noise allows the performance estimation of the global system.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme widely used in several communications standards such Digital Video Broadcast (DVB), Long Term (LTE) and Local Multipoint Distribution Service (LMDS). However OFDM is known to be highly sensitive to analog Radio-Frequency (RF) impairments [1][2][3]. RF building blocks impose several mismatches which may cause random variation on
amplitude and phase of the received signal [4]. Phase noise is one of the most studied RF impairment, since it causes unwanted phase variations in the oscillator signals used for frequency conversion. Main studies about phase noise are based on free run-oscillators [5][6]. However since the signal used for frequency conversion must have high accuracy in small and precise variations frequency steps, the oscillator is embedded in a synthesizer topology [7].

In single carrier modulation schemes it is possible to establish a direct relation between RF impairments based on spectrum analysis, namely phase noise effect, and present a direct relation with typical base-band (BB) figures of merit such as error vector magnitude (EVM), symbol error rate (SER) and bit error rate (BER) [4][8][9].

In OFDM modulation scheme phase noise will cause the orthogonality loss of the several sub-channels, leading to EVM degradation and consequently BER, on the receiver. The effect of phase noise on OFDM transmission can be divided into two effects. One is the rotation of the received constellation by an angle which is equal to the mean of the phase deviation of an oscillator called common phase error (CPE). The other effect is inter-carrier interference (ICI) which is caused by the fast changes of the oscillator phase. Typical OFDM system uses carrier pilots to estimate the propagation, time delay and frequency deviation between the transmitter and receiver. This is done before the RF conversion, therefore, these pilots will also be affected with LO phase noise.

2. RF Transceiver System

RF front-end has the major role on modulation and demodulation of analogue IQ signals, located between the antenna and the analog to digital (A/D) conversion. Regardless the BB signal format is, RF front end converts IQ signals, as depicted in Fig. 1. Therefore it is imperative to evaluate the behavior of the several building blocks on the IQ signals.

![Fig. 1. Transceiver architecture based on IQ signals](image)

The BB signals from the BB processor are up-converted by the mixer and the synthesizer output signal to the desired RF channel. The power amplifier (PA) amplifies the signal to the desired transmission power. The receiver is composed by the low noise amplifier (LNA) which amplifies the RF band in which the desired RF channel is located. The RF band is then down-converted by the synthesizer and mixer to an IQ BB signal, which is applied to the BB processor. This architecture is independent of BB signal composition, e.g. single carrier of OFDM, because both modulation techniques provide two orthogonal signals (IQ) to be converted.

The several RF building blocks impairments that imposes typical mismatch on the received signals are phase and amplitude imbalances, carrier phase and frequency synchronization errors, amplifier nonlinearities and phase noise [4][9]. Several of these impairments can be compensated on the BB processor through several of dedicated algorithms [10][11].

To obtain an accurate frequency for the mentioned conversions, the oscillator is based on frequency synthesis, as described in the following sections. In this work, since the main study is the phase noise, it is assumed that there is no IQ mismatch, the PA is operating in a linear zone and the combiner losses are negligible.
2.1. RF Synthesizer Topology

The oscillators used in RF transceivers are embedded in a synthesizer topology which is based on a negative loop feedback as in Fig. 2 [7].

![Fig. 2. Frequency synthesizer topology](image)

The presented architecture uses a low frequency reference \( f_{\text{REF}} \) which phase and frequency are compared with the signal obtained from the negative feedback loop. The phase-frequency detector (PFD) output signals are applied to a charge pump where the output voltage is proportional to the phase error and frequency error. The loop filter suppresses high frequency components from the output of the charge-pump, allowing the dc value to control the voltage controlled oscillator (VCO) frequency \( f_{\text{out}} \). The divider block \( M \) provides a low frequency signal which is compared with the reference signal.

With the presented architecture the signal output frequency of RF synthesizer changes with the \( M \) value as expressed (1).

\[
f_{\text{out}} = M \cdot f_{\text{REF}}
\]  

(1)

Synthesizer transient response is a nonlinear process and is not addressed in this work. However, a linear approximation can be used to obtain a trade-off in building blocks design [7].

2.1.1. Synthesizer Linear Model

The synthesizer linear model is presented in Fig. 3, where is presented, for each block, the corresponding transfer Laplace transfer function. The open and closed loop synthesizer transfer functions are given by (2) and (3), respectively.

![Fig. 3. Frequency synthesizer linear model](image)

\[
H_{\text{open}}(s) = \frac{K_d K_{\text{VCO}}}{M s^2} \cdot H(s)
\]  

(2)

\[
H_{\text{closed}}(s) = \frac{\phi_{\text{out}}(s)}{\phi_{\text{in}}(s)} = \frac{K_d K_{\text{VCO}} \cdot H(s) \cdot M}{M s^2 + K_d K_{\text{VCO}} \cdot H(s)}
\]  

(3)

Previous expressions are obtained in function of loop filter transfer function \( H(s) \). This is an important synthesizer building block, since it provides the stability, settling time and locking range. The main objective of this work is the phase noise study, therefore it is assumed that the synthesizer is stable and in lock conditions.
2.2. Phase Noise in RF Synthesizer

Phase noise is a widely studied phenomenon in free run oscillators [5][6]. The major influence of this impairment is the random phase variation of the VCO output voltage, as described in (4). The oscillator amplitude and frequency are described as $A_c$ and $f_c$ respectively. The small excess random phase variation, in a period, is represented by $\theta(t)$. In stable regime the oscillator operates in amplitude compression, therefore the random amplitude variation can be neglected [5][6].

$$c(t) = A_c \exp[j2\pi f_c t + \theta(t)]$$

(4)

Assuming that $|\theta(t)|<<1$ rad/s, (4) can be simplified in (5), which represents a narrow band phase modulation around the carrier.

$$c(t) = A_c [\cos(2\pi f_c t) - j\theta(t) \sin(2\pi f_c t)]$$

(5)

Phase noise is represented in frequency domain as shown in Fig. 4(a) which presents the carrier and the narrow band phase modulation effect caused the phase noise. Typical approaches quantifies phase noise for a given frequency carrier offset [5][6][7]. However this quantification is insufficient since it characterizes only one value for a given carrier offset frequency and does not describe the phase noise within a given bandwidth. This is an important characterization when the bandwidth of the analyzed sub-channels is compared with phase noise bandwidth.

![Diagram](image)

Fig. 4. Phase noise: (a) free run oscillator; (b) synthesizer in lock conditions

Synthesizer phase noise frequency response exhibits a low pass characteristic around the frequency carrier, since free run oscillator phase noise is modulated by (3), as presented in Fig. 4(b) [1][7]. The synthesizer phase noise model developed for this work is presented in 3.1.

3. System Model

OFDM is based on a multi-carrier modulation technique which uses orthogonal sub-carriers to transmit the BB data. The system model developed in MATLAB/Simulink is presented in Fig. 5.

Each transmitted symbol is described in discrete time representation, where the total number of samples is given by $N_{\text{amp}}$, corresponding to OFDM symbol, composed by $N$ samples, and cyclic prefix (CP) composed by $N_{\text{cp}}$. Pulse shaping is performed by a raised cosine described in discrete time domain.

For simplicity, it is assumed perfect frequency and timing synchronization between emitter and receiver, where the useful part of the received IQ signal samples can be expressed as (6). This signal is composed by samples of $m_I(t)$ and $m_Q(t)$.
Signals $s[n]$, $h[n]$ and $\xi[n]$ denote the samples of the useful part of the transmitted signal $x(t)$, of the channel impulse response $h(t)$ and the phase noise process $\theta_{err}(t)$ at the output of the synthesizer, respectively. The term $\xi[n]$ represents AWGN samples for a given SNR.

In the frequency domain, since the bandwidth of a sub-channel is designed to be smaller than the available channel bandwidth, each sub-channel is seen as a flat fading channel which simplifies the equalization process [3]. In the time domain, by splitting a high-rate data stream into a number of lower-rate data streams that are transmitted in parallel, OFDM resolves the problem of ISI in wide band communications.

The presented model allows the configuration of the sub-channel number, $N_{CH}$.

### 3.1. Synthesizer Phase Noise Model

As described in section 2.2, the synthesizer phase noise frequency response presents a low pass characteristic, where the correspondent developed noise model is presented in Fig. 6.

The output values of the Gaussian noise generator are parameterized by $v_{\text{noise}}^2$ and applied to a low pass filter, $H_{PLL}(s)$, with configurable gain and cut-off frequency, $f_{\text{cut}}$. This filter represents the synthesizer phase noise transfer function. The output signal, $LO_{PN}$, corresponds to the demodulation carrier with configurable phase noise power, $P_N$, and bandwidth [12].
By increasing the cut-off frequency, the power spectral density (PSD) decreases so that $P_N$ remains constant within $f_{\text{cut}}$ bandwidth. For this effect the $H_{\text{PLL}}(s)$ output signal is normalized with a corrective factor, corresponding to the integrated noise power spectral density (PSD) within $f_{\text{cut}}$. The normalization factor is given by

$$NORM = 2 \cdot f_{\text{cut}} \cdot \arctan\left(\frac{f_{\text{samp}}}{2 \cdot f_{\text{cut}}}\right)$$

This simulation feature allows the OFDM ICI study based on PSD for several sub-channel bandwidth and noise power. The global system evaluation is obtained by EVM.

3.1.1. Error Vector Magnitude

The relation between phase noise and BB figure of merit can be obtained with EVM. This metric is useful since it can, without decoding the channel content, quantify the symbol accuracy based on the ideal and distorted symbols, $s[n]$ and $r[n]$, respectively, as described by (8)[9].

Analyzing the IQ channels at the correct symbol timing on a scatter plot, the received constellation is obtained. The effect of phase noise impairments will be reflected in this graphical representation [9].

$$EVM = \sqrt{\frac{\sum_{k=1}^{N} (r[n] - s[n])^2}{(s[n])^2}}$$

Since the BB content is distributed by several sub-channels, EVM must be obtained for each one. Therefore, $EVM_{\text{OFDM}}$ is a given by $1 \times N_{\text{CH}}$ matrix composed by the correspondent EVM of each analyzed sub-channel.

$$EVM_{\text{OFDM}} = \begin{bmatrix} EVM_1 & EVM_2 & \ldots & EVM_{N_{\text{CH}}} \end{bmatrix}$$

Expression (9) presents the several EVM values for all the OFDM sub-channels. This allows the phase noise impact evaluation for each sub-channel.

4. Simulation Results

Unlike single carrier systems [9], OFDM spectral content does not present a direct relation with phase noise, due to CP and raised-cosine filtering. To obtain a significant result based on spectrum analysis, it is necessary to take into account the phase noise influence of the adjacent sub-channels on the studied one.

Due to OFDM channel specificities, phase noise leads to the loss of orthogonality between the $N_{\text{CH}}$ sub-channels. Phase noise power is far less compared with the sub-channel power. Therefore, the evaluation of synthesizer phase noise impact on the OFDM channel must be performed by canceling a given sub-channel and analyze the noise contribution of the remaining $N_{\text{CH}}-1$ sub-channels on it. This method allows the extraction of noise mean power on the cancelled sub-channel, corresponding to ICI.

Fig 7 presents the OFDM spectrum corresponding to a total of 30 sub-channels, 15 on the upper side band and 15 on the lower side band, where the sub-channel number 10 on the upper side band is cancelled. This is done by canceling a specific IFFT index given by $chSel$ presented in Fig 5. The ICI is obtained by repeating this process through all OFDM sub-channels. The following simulation results are performed for a variable number of sub-channels and phase noise bandwidth.
Fig. 7. Synthesizer simulation noise model

Fig. 8(a) presents the noise mean power variation along the OFDM channel bandwidth for a constant integrated phase noise power of -37dBc and variable synthesizer bandwidth, for 40 active sub-channels, corresponding to a sub-channel bandwidth of 33kHz. As depicted, when noise bandwidth increase, OFDM ICI increases, since the phase noise on the adjacent sub-channels will influence the measured one.

Fig. 8(b) presents, for a noise bandwidth of 1 kHz and phase noise power of -37dBc, ICI variation with the $N_{CH}$ sub-channels, e.g., with sub-channel bandwidth. As depicted, ICI decreases when the number of active sub-channels decreases.

Fig. 9 presents EVM OFDM evolution, as described by (9), for a parametric simulation of phase noise power and bandwidth values. It is possible to infer that the system present EVM variations along the several active sub-channels, showing that the spectrum disposition of these sub-channels must be take into account for high values of ODFM ICI, namely with the spectrum disposition of the carrier pilots.
5. Conclusions

The influence of the phase noise synthesizer on OFDM ICI was described. The validation method was performed with a developed OFDM simulation model that allows the synthesizer modeling based on the noise mask. The proposed method requires canceling of one sub-channel in order to evaluate phase noise influence of the remaining on the cancelled one. This is a recursive method since all sub-channels must be evaluated.

For a constant integrated phase noise power within a specific sub-channel bandwidth, the obtained results show that OFDM ICI increases with the phase noise bandwidth. The multiplicative property of phase noise is verified, since when the phase noise bandwidth is greater than the sub-channel bandwidth, ICI power is higher than the noise source. OFDM ICI decreases when the number of active sub-channels also decreases.

After the phase noise model is characterized, the final metric used for the system evaluation is based on EVM. The obtained results, denotes that this values varies along OFDM channel bandwidth, as denoted when ICI was evaluated. This is an important trade-off to be take into account, namely for the allocation of carrier pilots along the OFDM channel.

References

